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The problem of electrical response of a piezoelectric plate transducer

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Abstract . An attempt has been made to find out the electrical response emitted by a piezoelectric transducers in the form of a plate with rigid backing under certain time-dependent applied stress To obtain the electrical response, we have to couple the equations connecting the two fields – mechanical and electrical and the method of Laplace transform has been used to solve the problem In the case of periodic stress, the disappearance of the electrical response after removal of stress is found to be non-instantaneous

Keywords Piezoelectric transducers, electrical response, stress

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1. Introduction

Studies in the disturbances of a piezoelectric material from the stand point of mechanics of continuous media have been initiated by Redwood [1]. Several other researchers [2-5] have extended the work to find out the disturbances in piezoelectric substances under different boundary conditions.

The problem of investigating mechanical and electrical disturbances in the piezoelectric transducers in the form of a plate and a bar under various time-dependent input conditions is of great importance in the branch of acoustics, specially in the detection of ultrasonic wave and in underwater signalling [6, 7]. Researchers [8-10] have studied the electrical response emitted by such type of transducer due to shock-loaded stress and found that the response to be transient in character.

The authors here attempt to find out the electrical response emitted by a piezoelectric transducers in the form of a plate, with rigid backing under certain applied periodic stress and interestingly it is found that the response is not zero, even when, the zero of the periodic stress occurs.

2. Fundamental equations and boundary conditions

We have considered a piezoelectric transducer in the form of a plate and the current is produced due to the propagation of a stress wave along the x-axis of the plate, which is taken to be the thickness direction of the plate. To obtain the electrical response, we have to couple the equations connecting the two fields – mechanical and electrical. The displacement current i due to the stress wave as in [9] is given by

$$i = A' \frac{dD}{dt}, \tag{1}$$

where A' is the electroded area and D is the electrical displacement.

We may write [9]

$$D = \frac{1}{x} \int_0^x P(x)\, dx,$$

where x is the thickness of the plate and P is the electric polarization.

In accordance with [10], $P(x) = f\sigma(x)$, where f is a constant and σ is the mechanical tension in the direction of the thickness of the plate.

We get from eq. (1)

$$i = \frac{A'f}{x} \frac{d}{dt} \int_0^x \sigma(x,t)\, dx,$$

$$\bar{i} = \frac{A'f}{x} p \int_0^x \bar{\sigma}(x,p)\, dx, \tag{2}$$

where p is the Laplace transform parameter and $\bar{i}$ & $\bar{\sigma}$ are the Laplace transforms of i & σ respectively.

The constitutive relation between stress and strain [2, 3] is given by

$$\sigma = cs - eE$$

$$= cs - \frac{e}{\varepsilon}(D - P) \qquad [D = P + \varepsilon E], \tag{3}$$

where c is the elastic constant, e-the piezoelectric modulus and ε is the dielectric permitivity. Therefore,

$$\frac{\delta\sigma}{\delta x} = c\frac{\delta s}{\delta x} - \frac{e}{\varepsilon}\left(\frac{\delta D}{\delta x} - \frac{\delta P}{\delta x}\right) \tag{4}$$

From Gauss' law, since there is no free charge inside the transducer

$$\operatorname{div} D = 0. \tag{5}$$

Since propagation of the stress wave is assumed to be plane in character, differentiation with respect to y and z is zero, *i.e.*,

$$\frac{\delta D_y}{\delta y} \quad \frac{\delta D_z}{\delta z} = 0 .$$

Therefore, from eq. (5),

$$\frac{\delta D_x}{\delta x} = 0 .$$

Eq. (4) becomes

$$\frac{\delta\sigma}{\delta x} = c\frac{\delta s}{\delta x} + \frac{e}{\varepsilon}\frac{\delta P}{\delta x} ;$$

$$\frac{\delta^2\xi}{\delta t^2} = c\frac{\delta^2 \xi}{\delta x^2} + \frac{\rho ef}{\varepsilon}\frac{\delta^2 \xi}{\delta t^2}; \qquad \left[s = \frac{\delta\xi}{\delta x}, \quad \frac{\delta\sigma}{\delta x} = \rho\frac{\delta^2\xi}{\delta t^2} \right.$$

where ξ is the displacement along the x axis.

Taking Laplace transform we have

$$\frac{\delta^2 \bar{\xi}}{\delta x^2} - \frac{\rho}{c}\left(1 - \frac{ef}{\varepsilon}\right)p^2\bar{\xi} = 0 . \tag{6}$$

The solution of this equation is

$$\xi = Ae^{-px/\vartheta} + Be^{px/\vartheta} \tag{7}$$

where $\vartheta^2 = c/[\rho(1 - ef/\varepsilon)]$, ef/ε being assumed to be greater than 1, and A and B are amplitude factors to be determined from boundary conditions.

For simplicity in computation, a transducer with rigid backing [8, 11] is preferred. We assume the extremity $x = X$ to be rigidly backed. When the mechanical impedance z_2 in the material 2, is ∞, then $(\bar{\xi})_X = 0$, where $(\bar{\xi})_X$ denotes the value of $\bar{\xi}$ evaluated at $x = X$ and similar interpretations may be given for the other suffixes. The electrical impedance is assumed to be a resistance of R ohms. Then the boundary conditions may be formulated as follows :

$$(\bar{\xi})_X = 0, \tag{8}$$

$$(\bar{\xi})_0 = (\bar{\xi}_1)_0 , \tag{9}$$

$$(\bar{\sigma}_1)_0 = (\sigma)_0 . \tag{10}$$

3. Solution of the problem

The equation of the transducer is written as follows :

$$\bar{V} = -\bar{i}R = -\frac{A'f}{\omega} pR \int_0^x \bar{\sigma}(x, p)dx . \tag{11}$$

From eq. (8)

$$Ae^{-pX/\vartheta} + Be^{pX/\vartheta} = 0. \tag{12}$$

From eq. (9)

$$A + B = A_1 + B_1 \tag{13}$$

where A_1 and B_1 are the values of A, B in material 1, and $\overline{V}$ is the electrical voltage.

Again, using eq. (3), we have

$$\overline{\sigma}\left(1-\frac{ef}{}\right) = c\frac{\delta\overline{\xi}}{\delta x} - \frac{e}{\varepsilon}\overline{D}.$$

Using eq. (1),

$$\sigma = \frac{c\varepsilon}{\varepsilon - ef}\frac{\delta\xi}{\delta x} - \frac{e}{\varepsilon - ef}\frac{i}{pA'}$$

Then eq. (10) gives

$$-c'\frac{P}{\vartheta_1}A_1 + c'\frac{P}{\vartheta_1}B_1 = -c\frac{P}{\vartheta}A + c\frac{P}{\vartheta}B. \tag{14}$$

Also we get from the relation $\sigma = cs - eE$ we get

$$e\frac{\delta\overline{V}}{\delta x} = c\frac{\delta\overline{\xi}}{\delta x} - \overline{\sigma}.$$

Then utilising eq. (11), the voltage difference $\overline{V}$ across the sample is given by

$$\overline{V} = \frac{c(A+B)}{X/A'fpR - e}. \tag{15}$$

Eliminating B_1 from eqs. (12) to (14), we have,

$$A = \frac{2c'\vartheta A_1}{(c\vartheta_1 + c'\vartheta) - e^{-2pX/\vartheta}(c'\vartheta - c\vartheta_1)},$$

$$B = \frac{2c'\vartheta A_1}{(c'\vartheta - c\vartheta_1) - e^{2pX/\vartheta}(c\vartheta_1 + c'\vartheta)}.$$

Then from eq. (15),

$$\overline{V} = \frac{2c'cA_1\vartheta}{x/A'fpR - e}\left\{\frac{1}{(c\vartheta_1 + c'\vartheta) - e^{-2pX/\vartheta}(c'\vartheta - c\vartheta_1)}\right.$$

$$\left.(c'\vartheta - c\vartheta_1) - e^{+2pX/\vartheta}(c\vartheta_1 + c'\vartheta)\right\}$$

We take the applied mechanical signal is periodic in nature *i.e.*,

$$F = F_0 sin\omega t,$$

$$L(F_0 sin\omega t) = \frac{F_0\omega}{p^2+\omega^2} = \text{first term of } (\bar{\sigma}_1)_0$$

$$= -\frac{c'\varepsilon}{\varepsilon - ef}\frac{p}{\vartheta_1}A_1 .$$

Therefore,

$$A_1 = -\frac{\varepsilon - ef}{c'\varepsilon}\frac{\vartheta_1\omega F_0}{p(p^2+\omega^2)}$$

Expanding as in [1, 7] and taking inverse Laplace transform, we get

$$V = \vartheta_0\left[-cos\omega t/\omega^2 + \beta sin\omega t/\omega^3 + 1/\omega\right], \tag{16}$$

where ϑ_0 and β are constants containing material parameters of the problem.

Eq. (16) shows the electrical response emitted by the piezoelectric plate transducer under periodic stress.

4. Discussion

Chakraborty [8] and Roy [10] have shown that the electrical response emitted by the transducer due to a shock stress is transient in character, which agrees with the result obtained by Redwood [1]. Whereas, the investigations of the authors of this paper revealed that a periodic stress results a periodic response together with a constant one. As a consequence, even at the zero values of the periodic stress, the response is not zero. This signifies that, the response takes some time to come to zero value after the external stress has been removed. So the disappearance of the response after the removal of stress is non-instantaneous. The physical significance of this is that the crystal has some time-lag to come to its ground state.

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